Pensieve Header: The uniqueness of R in β calculus.

In[2]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-01"];
<< betaCalculus.m;
βSimplify = FullSimplify;
```

In[4]:=
```
{
   Rp = R[1, 2],
   Rm = RInv[1, 2],
   Rp ** Rm,
   Rp + (Rm // dP[1 → 3, 2 → 4]) // dm[1, 3, 1] // dm[4, 2, 2]
  } // βForm // ColumnForm
```

Out[4]=

$$\begin{pmatrix} W[1] & h[2] \\ t[1] & \frac{-1+e^{c[1]}}{c[1]} \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[2] \\ t[1] & \frac{-1+e^{-c[1]}}{c[1]} \end{pmatrix}$$

$$(W[1])$$

$$(W[1])$$

In[5]:=
```
{
    Rp = W[1] + Sum[ρ_{10 i+j}[c[1], c[2]] ar[i, j], {i, 2}, {j, 2}],
    Rm = W[1] + Sum[σ_{10 i+j}[c[1], c[2]] ar[i, j], {i, 2}, {j, 2}],
    t1 = Rp ** Rm,
    t2 = Rp + (Rm // dP[1 → 3, 2 → 4]) // dm[1, 3, 1] // dm[4, 2, 2]
  } /. α_[c[1], c[2]] :→ α // βForm // ColumnForm
```

Out[5]=

$$\begin{pmatrix} W[1] & h[1] & h[2] \\ t[1] & \rho_{11} & \rho_{12} \\ t[2] & \rho_{21} & \rho_{22} \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[2] \\ t[1] & \sigma_{11} & \sigma_{12} \\ t[2] & \sigma_{21} & \sigma_{22} \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[1] & h[2] \\ t[1] & \rho_{11} + (1 + c[1]\,\rho_{11})\,\sigma_{11} + \frac{c[2]\,\rho_{12}\,(1+c[1]\,\rho_{11}+c[2]\,\rho_{21})\,\sigma_{21}}{1+c[1]\,\rho_{12}+c[2]\,\rho_{22}} & \rho_{12} + \frac{(1+c[1]\,\rho_{11})\,(1+c[1]\,\rho_{12}+c[2]\,\rho_{22})\,\sigma_{12}}{1+c[1]\,\rho_{11}+c[2]\,\rho_{21}} + c[2]\,\rho \\ t[2] & \rho_{21}\,(1 + c[1]\,\sigma_{11}) + \frac{(1+c[1]\,\rho_{11}+c[2]\,\rho_{21})\,(1+c[2]\,\rho_{22})\,\sigma_{21}}{1+c[1]\,\rho_{12}+c[2]\,\rho_{22}} & \rho_{22} + \frac{c[1]\,\rho_{21}\,(1+c[1]\,\rho_{12}+c[2]\,\rho_{22})\,\sigma_{12}}{1+c[1]\,\rho_{11}+c[2]\,\rho_{21}} + \sigma_{22} + c[2] \end{pmatrix}$$

$$\begin{pmatrix} W\left[1 - \frac{c[1]\,c[2]\,\rho_{21}\,\sigma_{12}}{(1+c[1]\,\rho_{11}+c[2]\,\rho_{21})\,(1+c[1]\,\sigma_{12}+c[2]\,\sigma_{22})}\right] & h[1] \\ t[1] & \sigma_{11} + \rho_{11}\,(1 + c[1]\,\sigma_{11}) + \frac{c[2]\,(1+c[1]\,\rho_{11})\,\rho_{21}\,(1+}{1+c[1]\,\sigma_{12}+c[1]\,\rho_{11}\,(1+c[1]\,\sigma_{12}+c[2]\,\sigma_{22})} \\ t[2] & (1 + c[1]\,\rho_{11} + c[2]\,\rho_{21})\left(\sigma_{21} + \frac{\rho_{21}\,(1+c[1]\,\sigma_{11})\,(}{1+c[1]\,\sigma_{12}+c[1]\,\rho_{11}\,(1+c[1]\,\sigma_{12}+c[2]} \right. \end{pmatrix}$$

In[6]:=
```
eqns = Join[
   βEquations[t1 == W[1]],
   βEquations[t2 == W[1]]
  ] /. α_[c[1], c[2]] :> α
```

Out[6]=

$$\Big\{\rho_{11} + (1 + c[1]\,\rho_{11})\,\sigma_{11} + \frac{c[2]\,\rho_{12}\,(1 + c[1]\,\rho_{11} + c[2]\,\rho_{21})\,\sigma_{21}}{1 + c[1]\,\rho_{12} + c[2]\,\rho_{22}} == 0,$$

$$\rho_{21}\,(1 + c[1]\,\sigma_{11}) + \frac{(1 + c[1]\,\rho_{11} + c[2]\,\rho_{21})\,(1 + c[2]\,\rho_{22})\,\sigma_{21}}{1 + c[1]\,\rho_{12} + c[2]\,\rho_{22}} == 0,$$

$$\rho_{12} + \frac{(1 + c[1]\,\rho_{11})\,(1 + c[1]\,\rho_{12} + c[2]\,\rho_{22})\,\sigma_{12}}{1 + c[1]\,\rho_{11} + c[2]\,\rho_{21}} + c[2]\,\rho_{12}\,\sigma_{22} == 0,$$

$$\rho_{22} + \frac{c[1]\,\rho_{21}\,(1 + c[1]\,\rho_{12} + c[2]\,\rho_{22})\,\sigma_{12}}{1 + c[1]\,\rho_{11} + c[2]\,\rho_{21}} + \sigma_{22} + c[2]\,\rho_{22}\,\sigma_{22} == 0,\ \text{True},$$

$$\sigma_{11} + \rho_{11}\,(1 + c[1]\,\sigma_{11}) + (c[2]\,(1 + c[1]\,\rho_{11})\,\rho_{21}\,(1 + c[1]\,\sigma_{11})\,\sigma_{12})\,/\,(1 + c[1]\,\sigma_{12} + c[1]\,\rho_{11}\,(1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22}) + c[2]\,(\sigma_{22} + \rho_{21}\,(1 + c[2]\,\sigma_{22}))) == 0,$$

$$(1 + c[1]\,\rho_{11} + c[2]\,\rho_{21})\,(\sigma_{21} + (\rho_{21}\,(1 + c[1]\,\sigma_{11})\,(1 + c[2]\,\sigma_{22}))\,/\,(1 + c[1]\,\sigma_{12} + c[1]\,\rho_{11}\,(1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22}) + c[2]\,(\rho_{21} + (1 + c[2]\,\rho_{21})\,\sigma_{22}))) == 0,$$

$$((1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22})\,((1 + c[1]\,\rho_{11})\,(1 + c[2]\,\rho_{22})\,\sigma_{12} + \rho_{12}\,(1 + c[1]\,\sigma_{12} + c[1]\,\rho_{11}\,(1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22}) + c[2]\,(\rho_{21} + (1 + c[2]\,\rho_{21})\,\sigma_{22}))))\,/\,(1 + c[1]\,\sigma_{12} + c[1]\,\rho_{11}\,(1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22}) + c[2]\,(\sigma_{22} + \rho_{21}\,(1 + c[2]\,\sigma_{22}))) == 0,$$

$$(c[1]\,\rho_{21}\,\sigma_{12} + (1 + c[1]\,\rho_{11} + c[2]\,\rho_{21})\,\rho_{22}\,(1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22}) + (1 + c[1]\,\rho_{11} + c[2]\,\rho_{21})\,(1 + c[2]\,\rho_{22})\,\sigma_{22}\,(1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22}))\,/\,(1 + c[1]\,\sigma_{12} + c[1]\,\rho_{11}\,(1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22}) + c[2]\,(\sigma_{22} + \rho_{21}\,(1 + c[2]\,\sigma_{22}))) == 0,$$

$$1 - \frac{c[1]\,c[2]\,\rho_{21}\,\sigma_{12}}{(1 + c[1]\,\rho_{11} + c[2]\,\rho_{21})\,(1 + c[1]\,\sigma_{12} + c[2]\,\sigma_{22})} == 1\Big\}$$

In[9]:=
```
eqns /. {ρ12 -> (-1 + e^c[1])/c[1], σ12 -> (-1 + e^-c[1])/c[1], ρ_ -> 0, σ_ -> 0} // Simplify
```

Out[9]= {True, True, True, True, True, True, True, True, True, True}

In[11]:=
```
Reduce[eqns /. {ρ12 -> ρ, σ12 -> σ, ρ_ -> 0, σ_ -> 0}, {c[1], c[2]}]
```

Out[11]=

$$(\sigma == 0\ \&\&\ \rho == 0)\ ||\ \left(\rho\,\sigma \neq 0\ \&\&\ c[1] == \frac{-\rho - \sigma}{\rho\,\sigma}\right)$$

In[13]:=
```
Solve[eqns /. {ρ12 -> 1, σ12 -> σ, ρ_ -> 0, σ_ -> 0}, {σ}]
```

Out[13]=

$$\left\{\left\{\sigma \to -\frac{1}{1 + c[1]}\right\}\right\}$$

In[16]:=
```
{
   Rp = W[1] + ar[1, 2],
   Rm = W[1] - 1/(1 + c[1]) ar[1, 2],
   Rp ** Rm,
   Rp + (Rm // dP[1 → 3, 2 → 4]) // dm[1, 3, 1] // dm[4, 2, 2]
  } // βForm // ColumnForm
```

Out[16]=

$$\begin{pmatrix} W[1] & h[2] \\ t[1] & 1 \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[2] \\ t[1] & -\frac{1}{1+c[1]} \end{pmatrix}$$

$$(W[1])$$

$$(W[1])$$

In[11]:=
```
{
   Rp = W[1] + α ar[1, 2],
   Rm = W[1] - α/(1 + α c[1]) ar[1, 2],
   Rp ** Rm,
   Rp + (Rm // dP[1 → 3, 2 → 4]) // dm[1, 3, 1] // dm[4, 2, 2]
  } // βForm // ColumnForm
```

Out[11]=

$$\begin{pmatrix} W[1] & h[2] \\ t[1] & \alpha \end{pmatrix}$$

$$\begin{pmatrix} W[1] & h[2] \\ t[1] & -\frac{\alpha}{1+\alpha\, c[1]} \end{pmatrix}$$

$$(W[1])$$

$$(W[1])$$

In[1]:=
```
False && SolveAlways[eqns, {c[1], c[2]}]
```

Out[1]= False